Unemployment and productivity, slowdowns and speed-ups: evidence using common shifts

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Abstract

We investigate the controversial issue whether unemployment is related to productivity growth in the long run, using U.S. data in a framework of infrequent mean shifts. Tests find (endogenously dated) shifts in 1973/74 and 1994/95, system techniques indicate that the shifts are common features, and the implied long-run link between the two variables is negative. Therefore the secular decline of unemployment since the mid 1990s indeed stemmed from higher average productivity growth. The initial and final regimes are essentially equal, thus supporting theories that explain the productivity slowdown by a slow adoption process of IT with associated learning costs.

Keywords: productivity slowdown, growth, NAIRU level, common shifts

JEL codes: E24, C32, O40

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1 Introduction

Whether there exists a long-run connection between unemployment and productivity is controversial among economists. It is clear that the *level* of (trending) productivity cannot reasonably have a long-run influence on the (non-trending) unemployment rate. However, a much stronger hypothesis is that "neither the level or rate of change of productivity has any long-run effect on the unemployment rate" (Stiglitz, 1997, p. 7; see for example Ball and Mankiw, 2002, for similar statements). Such a claim is in conflict with several model classes, including standard search and matching models of unemployment,[1] but also models where wage setting depends on reservation wages which are partly backward-looking (for whatever reason, see below). In this paper we show that changes in productivity growth indeed have a prolonged impact on unemployment, but that in the 1990s the USA have returned to a regime of "normal" values of average growth and unemployment.

The contributions of this paper can be summarized in three main points: One consists of a simple model in which we highlight that the dependence of the reservation wage on lagged (real) wages is sufficient to explain the observed negative long-run link between unemployment and productivity growth.[2] This is an attractive result because in reality unemployment benefits are partially indexed to past wages, and therefore assuming such a link to past wages is completely natural.[3] Another point is that we provide further evidence on the existence of a long-run connection between productivity growth and unemployment. Using new co-breaking methods we confirm the findings in Pissarides and Vallanti (2007) or Tripier (2006), in contrast to the results of Muscatelli and Tirelli (2001) which were insignificant for the USA. And finally, in our bivariate system we can identify separate regimes and we show that the post-1994 equilibrium is essentially equal to the one of the pre-1974 period. In that sense the productivity slowdown and the associated

[1]The usual references are Aghion and Howitt (1994) and Mortensen and Pissarides (1998); see Prat (2007) for a recent interesting extension.

[2]Assuming such a backward-looking reservation wage goes back to Blanchard and Katz (1999). See Tripier (2006) for a richer model embedding the same feature.

[3]Hogan (2004) showed that such a link is also significant in British micro data.

elevated average unemployment rates are history. These findings support the view that the productivity slowdown was a prolonged but ultimately transitory phenomenon, induced by the slow diffusion of information technology (IT) with associated temporary adoption costs.

With respect to empirical methods, we start by using the univariate techniques developed in Bai and Perron (1998) and Bai and Perron (2003) to estimate the number and the dates of the breaks. However, the main empirical novelty of this paper is the system analysis to test whether the shifts identify a common structure. This approach uses the concept of "co-breaking" as introduced by Hendry (1997) (see also Krolzig and Toro, 2002, and Massmann, 2003), and is a special case of the notion of common features (Engle and Kozicki, 1993). The number of variables in a co-breaking analysis is conceptually limited by the number of shifts, which explains why there have not been many economic applications of co-breaking methods until now. However, since longer time series as well as methods to estimate shifts are becoming more widely available, the co-breaking approach promises to be a very useful tool for econometric modellers.

The remainder of this paper is structured as follows: The following section 2 contains a brief description of the univariate mean-shift tests and their results. Then section 3 formulates the statistical framework for the system analysis, explains the test and estimation procedures, and reports the system results. Afterwards section 4 presents a stylized and stripped-down theoretical model to highlight the long-run implications of backward-looking reservation wages. Finally, section 5 provides some conclusions.

2 Mean shifts in productivity growth and unemployment

We first apply the univariate methods developed by Bai and Perron (1998, 2003) that are capable of estimating and testing for the number of breaks and their location in the sample. This approach was applied by Hansen (2001) to labor productivity growth of the U.S. manufacturing sector, and by Papell et al. (2000) to annual data of the unemployment

rates of several OECD countries up to 1997. An earlier (less formal) finding of a break in U.S. unemployment in 1974 is given in Evans (1989). Because unemployment rates do not refer just to manufacturing, we accordingly use productivity growth of the entire business sector (quarterly, denoted by Δq_t), and compared to Papell et al. (2000) and Evans (1989) we use longer samples and higher-frequency data for unemployment rates, namely monthly series (u_t^m).[4]

The univariate framework for the multiple-shifts test is given by

$$x_t = \delta_j + \epsilon_{uni,t}, \quad t = T_{j-1} + 1, ..., T_j, \quad T_0 = 0,\ T_{m+1} = T, \tag{1}$$

for $j = 1, ..., m + 1$, where m is the number of shifts, so there are $m + 1$ regimes. In each regime there is a potentially different intercept δ_j, but the break points T_j are unknown. It is important to note that the residual $\epsilon_{uni,t}$ need not be white noise nor homogeneous; in the Gauss program that is provided by Perron on the website of the Journal of Applied Econometrics and that was used for these tests the covariance estimate accounts for that. We always allow for as much heterogeneity as possible. There are two differences between the applications to the two variables: We use the pre-whitening option for the residual covariance estimation for Δq_t but not for u_t^m, because with the near-unit-root serial correlation in u_t^m the pre-whitening procedure seemed to break down, producing erratic and unreliable test results. Also, we use a trimming value of 0.15 for Δq_t but 0.1 for u_t^m, because of the different sample sizes ($T = 226$ for Δq_t, $T = 673$ for u_t^m). The trimming value determines the minimal length of potential regimes; according to Bai and Perron (2003), when heterogeneity in the residuals is allowed but no serial correlation, a trimming of 0.15 is appropriate for $T = 120$, implying a minimal regime length of 18 observations. Our choices mean minimal regime lengths of 33 (for Δq_t) or 67 (for u_t^m) observations while also allowing for serial correlation.

[4]Productivity growth Δq_t is measured as the first time difference of quarterly log hourly real labor productivity of the business sector (seasonally adjusted), the source is the Bureau of Labor Statistics (BLS), sample 1948q1-2003q3. Unemployment is a survey-based (16 years and over) seasonally adjusted measure in percentage points also provided by the BLS.

The description of the global minimization algorithm for the break date estimation in Bai and Perron (2003) will not be reproduced here, as we essentially use it as a black box. However, it is necessary to introduce the various test hypotheses and respective statistics. All their distributions are non-standard.

First there are supremum-type test statistics for the fixed null hypothesis $H_0 : m = 0$ against various fixed alternatives: $H_{1,j} : m = j$ for $j = 1, ..., m_{max}$, where we set $m_{max} = 5$. Obviously, the power of each of these tests will depend on the true number of breaks. Also they are not independently distributed, but nominal significance of at least one of these tests may point to the existence of at least one break. Let us denote these statistics by $supF(j|0)$. Next, $UDmax$ and $WDmax$ are two related statistics that do not specify the precise number of breaks under the alternative, thus $H_0 : m = 0$ and $H_1 : 1 < m \leq m_{max}$. They differ in their weighting schemes.

Finally, and most importantly, Bai and Perron (2003) suggest a testing sequence that in effect estimates the number of breaks. The most restricted model of course imposes $m = 0$ which is the starting point. Then the procedure partitions the sample according to all previously found breaks and then tests for single additional breaks in each subsample. The procedure stops at the first non-significant test. For $j = 1, ..., m_{max}$ the nested sequence of hypothesis pairs is then $H_{0,j} : m = j - 1$ against $H_{1,j} : m = j$. Let us call these statistics $supF(j|j-1)$.

Another possible approach would be to use information criteria for model selection to determine the number of breaks. However, they severely overestimate the number of breaks with serially correlated errors and do not allow for residual heterogeneity across regimes and are therefore not attractive.

In table 1 we report the test results displaying only the significance level, not the actual numbers and critical values which would be distracting and not really helpful. The tests clearly indicate that there are some mean shifts in the variables over the analyzed samples. For productivity growth Δq_t the sequence clearly indicates two breaks, while for u_t^m even

Table 1: Tests for number of mean shifts in the variables

	$UDmax$	$WDmax$	$supF(1\|0)$	$supF(2\|0)$	$supF(3\|0)$
Δq_t	*	**	**	**	**
u_t^m	***	***	**	***	***

	$supF(2\|1)$	$supF(3\|2)$	$supF(4\|3)$
Δq_t	**	n.s.	-
u_t^m	***	***	n.s.

	sample	T	trimming
Δq_t	1948q1-2003q3	226	0.15
u_t^m	1948m1-2004m1	673	0.10

Notes: Upper panel tests for existence of at least one break, lower panel has sequence of tests to determine the number. Significance denoted by * (10%), ** (5%), *** (1%), or "n.s." (p-value greater than 10%). To avoid clutter, $supF(4|0)$ and $supF(5|0)$ are not reported. "q" and "m" stand for quarter and month, respectively. For the meaning of the trimming value see the text.

three breaks are found. We will address the issue of the "extra" break in unemployment in the system context below, but at this point we already conjecture that the near-unit-root dynamics of unemployment may bias the test sequence somewhat towards finding more mean shifts than are actually present. The intuition for our conjecture is that in a nearly integrated process a large shock will shift the (conditional) mean for quite a long time, such that this may easily be mistaken with a shift of the unconditional mean.

The resulting break date and intercept estimates for the various regimes are collected in table 2. Most importantly, at the end of the 1990s the means of both variables return to their original pre-1970s values. Secondly, the dates of the first and last breaks roughly coincide across the two variables after accounting for estimation uncertainty, which is a requirement for common shifts. However, while the break dates are estimated quite precisely for the unemployment rate, there is considerable uncertainty for productivity growth. It is also interesting that with this economy-wide data we are able to formally confirm the stylized fact of the productivity slowdown after 1973, whereas Hansen (2001) only found a single break in the 1990s (for manufacturing productivity data) or additional

Table 2: Estimated means and their break dates

	productivity growth, Δq_t	unemployment rate, u_t^m
first regime	3.24 (0.36)	4.8 (0.20)
	65q4←1973q1→84q3	74m6←1974m10→76m6
intermediate regime	1.40 (0.32)	7.6 (0.14)
	-	85m4←1986m10→88m5
intermediate regime 2	-	6.3 (0.17)
	88q3←1995q2→end	94m2←1994m10→96m7
last regime	3.12 (0.44)	5.0 (0.10)

Notes: "q" and "m" stand for quarter and month, respectively. Numbers in parentheses are standard errors. The unemployment rate is given in percentage levels, and the productivity growth in annualized percentage rates. The estimates and 95% confidence intervals for the break dates ("←→") were calculated with the Gauss program by Perron mentioned in the text.

breaks in 1964 and 1982 (for the industrial production index divided by hours worked). With respect to the unemployment rate we find the same breaks as was possible for Evans (1989) and Papell et al. (2000) with their respective samples, but find an additional break in 1994. Note again that these estimates will be reconsidered within the system context.

3 System analysis of common shifts

Since the used variables are observed at different frequencies (quarterly vs. monthly), we first converted the monthly unemployment rate to quarterly data by averaging the respective observations, and we denote the quarterly series simply by u_t.[5]

3.1 The statistical framework

To analyze the relation between the breaks in the variables we use a vector-autoregressive (VAR) model augmented with shift dummies. Although there are some important differ-

[5]The empirical analysis was performed with PcGive (Doornik and Hendry, 2001) and batch programs written in Ox (Doornik, 2002).

ences it will be helpful to keep in mind the analogy to a standard cointegrated VAR, see Johansen (1995). The common shifts in our model replace the common stochastic trends driving the $I(1)$ variables in a cointegration model. The model we use is a special case of the ones discussed in Krolzig and Toro (2002) and Massmann (2003), who in turn draw from Hendry (1997), and is given by:

$$\mathbf{x_t} = \sum_{i=1}^{p} \Gamma_i \mathbf{x_{t-i}} + \kappa + \Psi \mathbf{s_t} + \Phi \mathbf{d_t} + \epsilon_{\mathbf{multi,t}}, \quad t = 1, ..., T \tag{2}$$

Here $\mathbf{x_t}$ is the $(n \times 1)$-variable vector, where in our case $\mathbf{x_t} = (\Delta q_t, u_t)'$, p is the lag length, κ is a constant term and thus the intercept of the first regime before any breaks happen. As in the previous section, m is still the number of (common) shifts, and the break points are still denoted by T_j $(j = 1, ..., m)$. Thus $\mathbf{s_t} = (1(T_1 < t \leq T_2), ..., 1(T_m < t \leq T))'$ is the $(m \times 1)$-vector of additional intercepts for regimes $2, ..., m + 1$, using the standard indicator function $1(.)$ that takes the value of one if its argument is true, else zero. Ψ is a $(n \times m)$-coefficient matrix. The vector $\mathbf{d_t}$ collects a number of impulse dummies $(\{..., 0, 0, 1, 0, 0, 0, ...\})$ as described in the concrete specification below, and finally $\epsilon_{\mathbf{multi,t}}$ is a white noise error term.

The common shifts are assumed to be the only source of non-stationarity, so we impose that the roots of $I - \sum_{i=1}^{p} \Gamma_i z^i$ are all stable. We first note that if $n > m$, i.e. if we have more variables than breaks, then there will always exist a linear combination of variables without shifts. This is called "spurious co-breaking" and does not allow any meaningful interpretation.[6] Thus we will restrict ourselves to the situation $n \leq m$, where it is clear that we have $\text{rank}(\Psi) \leq n$. But if $\text{rank}(\Psi) = n$ the shifts in the variables are linearly independent and thus no co-breaking can occur. Hence the condition for the existence of linear combinations of variables that are free from shifts is given by

$$b \equiv \text{rank}(\Psi) < n, \tag{3}$$

[6]However, the term "spurious" is perhaps a little too strong, because the fact that the break dates of the variables coincide already contains valuable information and is by no means automatic.

which means that the proportions of the different mean shifts in the variables are linearly related. The "co-breaking rank", i.e. the number of linearly independent stationary combinations of the variables, is $n - b$. If Ψ has reduced rank b, then it may be decomposed as

$$\Psi = \eta\xi', \tag{4}$$

where η and ξ are two $(n \times b)$-matrices, each of full rank b. As is well known from cointegration analysis, this decomposition is only unique after imposing normalizing and identifying restrictions. The matrix ξ determines the proportions of the various shift dummies in each "shift package", while η loads the different shift packages into the variables. After taking into account the lag polynomial the long-run impact of the shifts $\xi'\mathbf{s_t}$ is

$$\eta^* = \left(I - \sum_{i=1}^{p} \Gamma_i\right)^{-1} \eta, \tag{5}$$

so the estimated shifting means in the variables are altogether given by

$$\left(I - \sum_{i=1}^{p} \widehat{\Gamma}_i\right)^{-1} (\widehat{\kappa} + \widehat{\eta}\widehat{\xi}'\mathbf{s_t}). \tag{6}$$

We denote by $\beta_\perp$ any orthogonal complement of a given $(n \times b)$-matrix β with full column rank.[7] Multiplying with $\eta_\perp^{*'}$ then removes the common shifts because $\eta_\perp^{*'}\eta^*\xi'\mathbf{s_t} = 0$. Therefore we call $\eta_\perp^*$ the co-breaking matrix and $\eta_\perp^{*'}\mathbf{x_t}$ is stationary. With a view to our specific application we note that in the bivariate case with two breaks and under co-breaking ($n = 2,\ m = 2,\ b = 1$) the matrix $\eta_\perp^{*'}$ is actually a (1×2)-vector. To aid interpretation we will normalize $\eta_\perp^*$ such that the second element is unity, which we denote by $\eta_{\perp(2)}^* = 1$. Then the equilibrium relation between the variables that is identified

[7] The matrix $\beta_\perp$ has dimension $n \times n - b$, full column-rank, is a basis of the null space of β, and satisfies $\beta_\perp'\beta = 0$.

by the fact that the deviations are stationary and do not contain shifts can be written as:

$$u_t = -\eta^*_{\perp(1)}\Delta q_t + \epsilon^*_t, \tag{7}$$

where ϵ^*_t are the resulting stationary fluctuations around the equilibrium.

It is therefore essential for the analysis of common shifts to determine the rank of Ψ. Given that no integrated variables are involved this is a standard reduced rank regression problem. We write the system in matrix form as

$$X = A\Gamma + S\Psi + E_{multi}, \tag{8}$$

where X is $T \times n$, the $(T \times pn + 1 + \text{rows of } \mathbf{d_t})$-matrix A collects the observations on lags, the constant, and the impulse dummies, Γ is the corresponding coefficient matrix, S is the $T \times m$ data matrix of the shift dummies, and Ψ is as before. As in the Johansen procedure, the unrestricted terms in A are first concentrated out:

$$R_0 = (I_T - A(A'A)^{-1}A')X, \qquad R_1 = (I_T - A(A'A)^{-1}A')S \tag{9}$$

Then the moment matrices are formed:

$$M_{ij} = T^{-1}R_i'R_j \quad i,j = 0,1 \tag{10}$$

And we have $M_{01} = M_{10}'$. Then we solve the generalized eigenvalue problem corresponding to:

$$\lambda_i M_{11} v_i = M_{10} M_{00}^{-1} M_{01} v_i, \tag{11}$$

where λ_i are the eigenvalues with respective eigenvectors v_i. The number of (significant) non-zero eigenvalues determines the rank of Ψ, and the likelihood ratio test statistic for

the rank is

$$LR_b = -T \sum_{i=b+1}^{n} log(1-\lambda_i) \tag{12}$$

for the sequence of nested null hypotheses $H_{0,j-1} : b = j-1$ $(j = 1, ..., n)$, starting with the most restricted model $b = 0$. These statistics are asymptotically distributed as $\chi^2_{(n-b)(m-b)}$, and we stop at the first hypothesis that cannot be rejected. Given the resulting estimate $\hat{b}$ we can estimate ξ as the eigenvectors $V = (v_1 : ... : v_{\hat{b}})$ corresponding to the $\hat{b}$ largest eigenvalues λ_i $(i = 1, ..., \hat{b})$. As usual we can pick a just-identified estimate as $\hat{\xi} = V(\gamma' V)^{-1}$, where $\gamma' = (I_{\hat{b}} : 0_{n-\hat{b}})$. Finally, for a given $\hat{\xi}$ the estimation of η, Γ_i and by implication η^* along with some $\eta^*_\perp$ is a straightforward regression problem.

3.2 Empirical results

In our case, $n = 2$ and $m = 3$ initially, where however the status of the "extra" break in 1986 that we only found in the unemployment rate remains to be investigated. The concrete specification of model (2) is as follows: According to the previous estimates we set $T_1 = 1973q1$, $T_2 = 1986q4$, and $T_3 = 1994q4$. The reason for impulse dummies in $\mathbf{d_t}$ is threefold: First, $\Delta\mathbf{s_t}, ..., \Delta\mathbf{s_{t-p}}$ are used to effectively remove the post-break transition period from the sample. Second, the slight discrepancies between the point estimates of the break dates of the previous section are also "dummied out" by filling the gaps with impulse dummies. Finally, two outliers in the productivity equation are removed by individual dummies (1960q2 and 1951q3).[8] The effective sample is 1950q3-2003q3, $T = 213$. Setting the lag length to $p = 3$ proved necessary and sufficient to account for the dynamics of the variables. Diagnostic tests of this specification are shown in table 3. It can be seen that there are ARCH effects in the unemployment equation,

[8] Altogether this yields impulse dummies for the following observations initially: 1951q3, 1960q2, 1973q1, 1973q2, 1973q3, 1973q4, 1974q1, 1974q2, 1974q3, 1974q4, 1975q1, 1975q2, 1986q4, 1987q1, 1987q2, 1987q3, 1987q4, 1994q4, 1995q1, 1995q2, 1995q3, 1995q4, 1996q1, 1996q2. So we lose a little more than 10% of the observations in each equation. However, many dummies are actually insignificant and can be deleted later on.

Table 3: Diagnostics of the initial system

	Δq_t-equation	u_t-equation
no autocorr. (5 lags)	.27	.11
normality	.18	.00
no ARCH (4 lags)	.10	.00
no heterosc.	.91	.33
no vector autocorr. (5 lags)	.12	
vector normality	.00	
no vector heterosc.	.997	

Notes: Numbers are p-values. The system is described in section 3.1 in connection with the model (2).

which explains that the residuals are tested as non-normal. However, the general test for heteroscedasticity seems unaffected. We will try to account for the ARCH finding by also considering heteroscedasticity-robust covariance matrix estimates. The rank test on Ψ, however, will implicitly be based on a quasi-maximum-likelihood approach, given the non-normality of the residuals.

First we tried to simplify the system by checking the significance of the "extra" break in 1986 in the system context. The corresponding restriction that the intercepts from 1973-1986 and 1986-1994 are equal yields a χ^2_2 statistic of 1.80, p=0.41 (or 2.42, p=0.30, using the heteroscedasticity and autocorrelation consistent –HAC– covariances), and thus is clearly acceptable. In contrast, the analogous test of equal intercepts in 1986-1994 and 1994-2003 (i.e. no break in 1994) is clearly rejected with 12.2, p=0.00 (or 21.2, p=0.00 using HAC covariances). Despite the strong nominal significance of the 1986 break in the univariate analysis for unemployment it appears to be redundant in this system. Therefore we can remove the 1986q4-shift dummy along with its (lagged) differences from the system, and respecify the second regime as spanning the period 1973-1994.[9] This of course is good news from the perspective of a common-shifts analysis, and we will proceed with

[9]The joint restriction of removing also the corresponding impulse dummies yields a χ^2_{12} statistic of 7.19 (p=0.84). Using the HAC covariances produces the stunning value of 1100 (p=0.00), but since the residual variances during impulse-dummy-episodes are trivially estimated as close to zero, this result is clearly spurious.

the model $n = m = 2$, $T_1 = 1973q1$, $T_2 = 1994q4$.[10]

Note that after having established that there are two breaks in the system, the maximal number of variables for a meaningful analysis of common shifts is also two, given the arguments of section 3.1. Therefore, it would be impossible to add other covariates to the system that may be interesting on economic grounds.

Another indication that the restriction of no additional break in 1986 did not introduce any spurious non-stationarity is provided by the (inverse) roots of the lag polynomial, the largest of which is 0.76 (it is a real number) and thus much smaller than unity. This also confirms our assumption that the variables –and specifically the unemployment rate– are quite persistent, but do *not* contain unit roots.

With this model we applied the described rank test procedure to determine $\hat{b}$. The surprisingly clear-cut results are reproduced in table 4 and imply that the rank of Ψ is obviously $\hat{b} = 1$ and is thus reduced. The fact that $b > 0$ confirms that there are significant shifts in the data, and $b < n = 2$ implies that these are related in a linear fashion. Furthermore, the normalized estimate $\hat{\xi} = (1, 0.032)'$ suggests that only the regime 1973-1994 has an intercept different from the starting regime 1950-1973. The post-1994 additional intercept is negligible in comparison. This finding is also formally testable in a straightforward way since it just concerns the exclusion of $1(T_2 = 1994q4 < t)$ in the system, which produces a χ^2_2 statistic of 0.11, p=0.95 (0.16, p=0.92, using HAC covariances).[11] Now it is also clear why the second null hypothesis of the rank test ($H_0 : \ b = 1$) has a p-value close to unity: Given the reported clear-cut test result that the intercepts of the final and starting regimes are essentially equal, this means that implicitly we are dealing with just two effective regimes; the first (possibly "normal") regime is interrupted by the second (possibly "exceptional") one in the period 1973-1995. Recalling that the number of regimes is $m + 1$ and that in the situation of $m < n$ co-breaking is automatically achieved, it is clear

[10] We keep the impulse dummies that fill the slight gaps between the estimated break dates as discussed before; however, the impulse dummies for 1975q2, 1995q4, 1996q1, and 1996q2 were also completely redundant and were removed. Thus we keep 15 different impulse dummies altogether.

[11] After having removed this shift dummy, the exclusion of the corresponding impulses yields a χ^2_8 statistic of 7.20, p=0.52. Again, the HAC covariances are heavily biased, producing a spurious 1180, p=0.00.

Table 4: Testing for common shifts

H_0	eigenvalue	test stat. LR_b	d.o.f.	eigenvector
$b = 0$	0.11	25.0, p=0.00	4	$(1, 0.032)'$
$b = 1$	$6.6 * 10^{-8}$	$1.5 * 10^{-5}$, p=0.997	1	-

Notes: The rank test procedure is described in section 3.1. The used model has $n = m = 2$. The eigenvector is an estimate of ξ and describes the proportion of the regime-specific intercepts.

that with $m + 1 = n = 2$ the test statistic for a reduced rank of Ψ would be exactly equal to zero. However, note that equality of the intercepts of the starting and final regimes was never imposed before the tests, so this result is far from being spurious. Rather, it is an endogenous outcome of the data characteristics. The only thing to bear in mind is that the reported test result of exclusion of $1(T_2 = 1994q4 < t)$ logically implies the result of the test for reduced rank of Ψ.

Finally, we use this restricted model to estimate the co-breaking vector as described above; the result is that $\eta_{\perp}^{*\prime}\mathbf{x_t} = 1.12\Delta q_t + u_t$ is stationary and contains no shifts, and we can write the steady-state relation between annual productivity growth (in percentages) and unemployment as:

$$\overline{u} = 8.4 - 1.12\overline{\Delta q} \tag{13}$$

In terms of the model in section 4 the estimate 1.12 corresponds to $\mu\lambda/\beta_u$, and if we assume $\mu\lambda = 1$ for the United States due to the findings discussed in Blanchard and Katz (1999) we obviously would have an estimate $\hat{\beta}_u = 0.89$. The system estimates of the long-run means $(I - \sum_{i=1}^{p}\Gamma_i)^{-1}\kappa$ and additional means of the 1973-1994 regime $(I - \sum_{i=1}^{p}\Gamma_i)^{-1}1(T_1 = 1973q1 < t \leq T_2 = 1994q4)$ are given in table 5. Finally, the two variables are shown in figures 1 and 2, together with their estimated broken deterministic mean functions of the univariate and system approaches.

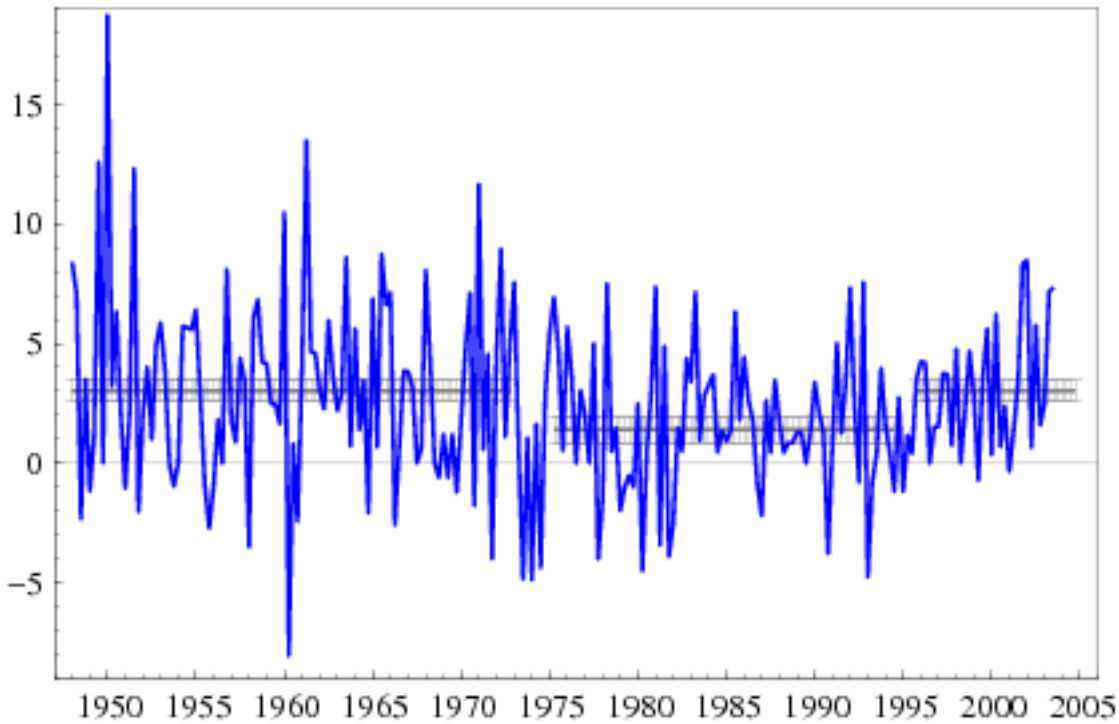
Of course the reaction of unemployment to productivity growth in the short run can be

Table 5: System estimates of regime-wise intercepts

	1950-1973 & 1995-2003	1974-1994
productivity growth	3.0 (0.23)	1.4 (0.29)
unemployment rate	4.8 (0.33)	6.8 (0.41)

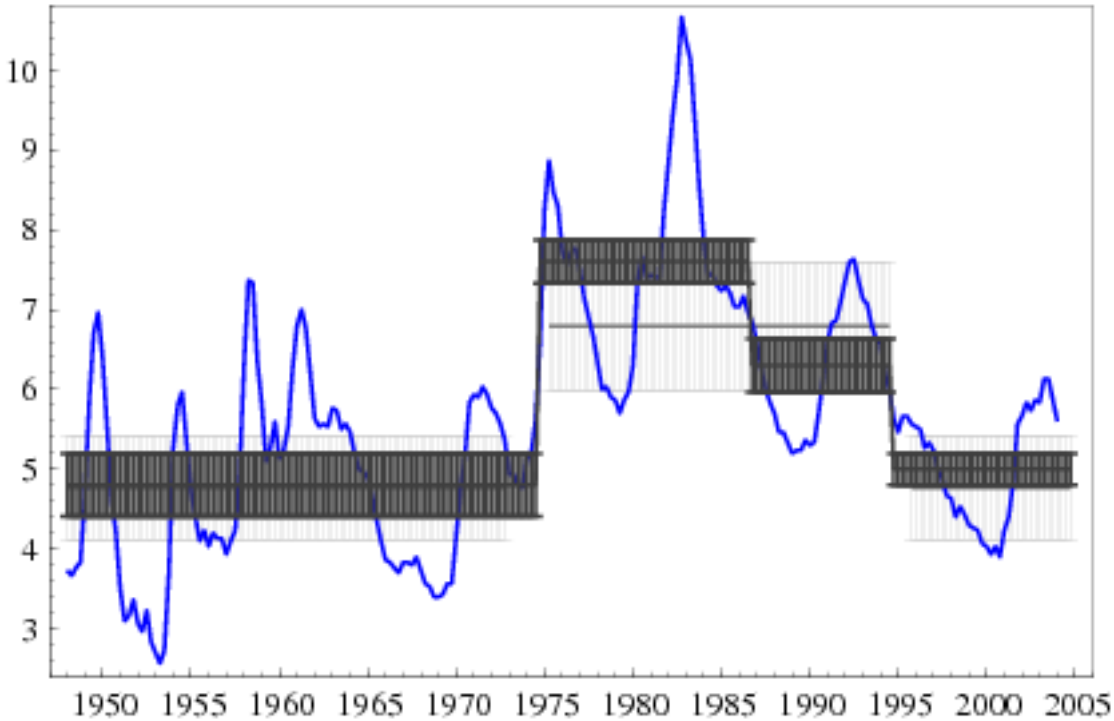
Notes: Estimates based on the final (restricted) VAR model; these long-run estimates correspond to (6) in terms of the system framework. Standard errors in parentheses.

Figure 1: Estimates of productivity growth regimes



Notes: This figure shows the shifting means of the annualized productivity growth rate as estimated in the final system described in section 3. The bands around the estimates indicate the 95% confidence intervals of the estimate based on standard errors in table 5. The estimates of the univariate procedure of Bai and Perron (1998, 2003) described in section 2 are almost identical and are omitted to avoid clutter. The gaps between the regimes reflect the transition phases that are left unmodelled.

Figure 2: Estimates of unemployment rate regimes



Notes: This figure compares the estimates of the shifting means of the unemployment rate of the univariate procedure of Bai and Perron (1998, 2003) in section 2 (black, with three breaks) and the system specification of section 3 (transparent grey, with two breaks). Bands around the estimates indicate 95% confidence intervals based on standard errors in tables 2 and 5. The gaps between the regimes reflect the transition phases that are left unmodelled.

radically different from what relation (13) suggests for the long run; for example, the most recent observations for both variables are above their estimated long-run means. Provided that no further shift has occurred, both variables would therefore be expected to fall over the medium term.

4 A stripped-down model of steady-state unemployment

In this section we present a bare-bones theoretical example in which the steady-state level of the labor market tightness depends on the growth rate of productivity, assuming reservation wages that partially depend on past real wages. The analysis focuses on the labor market and is partial equilibrium in the sense that technical progress as well as human and physical capital accumulation are not endogenized. By no means do we claim that this model is new or that it is superior to other theories which already exist (e.g., Tripier, 2006). It is also clear that partially backward-looking reservation wages are *not necessary* for a long-run link between productivity growth and unemployment, see Prat (2007) and the references therein (including standard search-matching models). However, an important institutional reason for incorporating the dependence of reservation wages on its own past into a model is that in all states of the USA unemployment benefits are tied to past wages (albeit to varying extents), see U.S.-Government (2004, p. 12). Since unemployment benefits influence the cost of search in matching models, and the outside option in bargaining models, there is a direct link to reservation wages.

The unemployment rate is of course not a perfect indicator for labor market tightness. However, other conceivable measures also suffer from various weaknesses: For example, the well-known „Help-wanted advertising index" is subject to various structural changes as described in Valletta (2005). Most notably the rise of the internet makes reliance on newspaper ads to measure trends in labor market tightness in recent years seem awkward. The Bureau of Labor Statistics' "job openings and labor turnover survey" (JOLTS) is only available from 2000 onwards. Also, while observed unemployment rates are in principle

also influenced by changes of labor force participation rates, a look at the data reveals that this cannot explain the pronounced shifts that we find (see e.g. Bradbury, 2005).

4.1 Production and labor demand

We assume a "right-to-manage" framework which means that firms can unilaterally determine how much labor they employ at the going wage rate (as opposed to bargaining over wages and employment levels simultaneously). This implies that the labor market outcome will always lie on the labor demand curve. This labor demand is derived from a constant-returns-to-scale production function of the standard constant-elasticity-of-substitution (CES) form:

$$Y = \gamma \left(\delta L_E^{(\sigma-1)/\sigma} + (1-\delta) K^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}, \tag{14}$$

where Y is aggregate output, L_E are effective labor inputs, and K is capital. Parameters are denoted by Greek letters; $\gamma > 0$ is the scale parameter, $\delta \in (0;1)$ is the distribution parameter, and $\sigma > 0$ is the elasticity of factor substitution; for $\sigma = 1$ we obtain the special case of the Cobb-Douglas function. Allowing for imperfect competition both in factor and output markets is straightforward as long as the respective mark-ups are assumed constant or at least stationary.

We adopt the interpretation from Durlauf and Quah (1999) that labor is enhanced with embodied human capital H and (disembodied) Harrod-neutral technological progress A, which are subsumed under $E \equiv AH$, such that E is a joint enhancement term. Note that the growth of E may slow down (or speed up) even when dA/dt is constant, depending on variations in the accumulation or depreciation speed of human capital. This distinction may become relevant for example in the story of Greenwood and Yorukoglu (1997). Altogether, effective labor inputs (L_E) therefore depend on hours worked (L) and

the described enhancement factor:

$$L_E = EL \tag{15}$$

Standard derivations yield the following implicitly defined labor demand equation (conditional on product demand Y):

$$q_t = c_{ld} + \sigma w_t + (1-\sigma)e_t + \epsilon_{ld,t}, \tag{16}$$

where lower-case latin letters denote logarithms and we have attached time indexes to the variables; $q_t \equiv y_t - l_t$ is observed hourly labor productivity, and w_t are real hourly labor costs. We will often refer to w_t simply as "wages". The constant c_{ld} contains various parameters (elasticity and mark-up terms, for example) and is not directly important here. Finally, $\epsilon_{ld,t}$ is a stationary error term with unconditional mean zero.

4.2 Wage setting

A convenient summary of a wage setting relation is given in Blanchard and Katz (1999):

$$w_t = \mu r_t + (1-\mu)q_t - \beta_u u_t + \epsilon_{w,t},\ 0 \leq \mu \leq 1, \beta_u > 0 \tag{17}$$

This states that real wages w_t are a function of the reservation wage r_t (including unemployment benefits etc.), the observed labor productivity q_t, and the (inverse) tightness of the labor market u_t. As discussed by Blanchard and Katz (1999) this formulation is compatible with bargaining models (including insider-outsider models) as well as with models that incorporate matching frictions, or efficiency wage considerations. Note that this wage function is linearly homogeneous in the reservation wage and in productivity, such that wages and productivity may grow at the same rate in the long run even with $\mu \neq 0$.

We follow Blanchard and Katz (1999) one step further and also posit that the reservation wage depends on past real wages and on current productivity levels in a linear-homogeneous fashion:

$$r_t = c_r + \lambda w_{t-1} + (1-\lambda) q_t, \; 0 < \lambda \leq 1 \tag{18}$$

Equation (18) is not only an intuitively plausible assumption due to the mentioned institutional characteristics of unemployment benefits, but is also supported by recent micro evidence: Hogan (2004) finds that the elasticity of the reservation wage with respect to past wages is significantly positive in household panel data for the UK, with a point estimate of about 0.3. Inserting (18) into (17) yields:

$$\Delta w_t = \mu c_r - (1-\mu\lambda)(w_{t-1} - q_{t-1}) - \beta_u u_{t-1} - \beta_u \Delta u_t + (1-\mu\lambda)\Delta q_t + \epsilon_{w,t} \tag{19}$$

So we see that the (lagged) labor share $(w-q)_{t-1}$ in general enters the wage setting curve, which is the major difference with respect to an expectations-augmented (wage) Phillips curve.[12]

4.3 Equilibrium

Combining the standard wage setting curve (19) with the labor demand function (16) yields:

$$\Delta w_t \;=\; c_{eq} - \frac{\alpha - \mu\lambda}{\alpha}(w_{t-1} - e_{t-1}) - \frac{\beta_u}{\alpha} u_t + \frac{\alpha - \mu\lambda}{\alpha}\Delta e_t + \epsilon_{eq,t} \tag{20}$$

with $\alpha \equiv 1 - \sigma(1-\mu\lambda)$, $c_{eq} \equiv (\mu c_b + (1-\mu\lambda)c_{ld})/\alpha$ and $\epsilon_{eq,t} \equiv (\epsilon_{w,t} + (1-\mu\lambda)\epsilon_{ld,t})/\alpha$.

[12] However, from an econometric point of view it is unfortunate that Blanchard and Katz (1999) and OECD (1997) call this an "error correction term" because this implicitly assumes that the labor share is stationary, which is clearly not the case in many countries. In the general case, covering non-stationary labor shares as well as unemployment rates, the actual error correction term is given by $w_{t-1} - q_{t-1} + \beta_u/(1-\mu\lambda)u_{t-1}$.

The term in square brackets is another equilibrium correction term, now written in terms of the unobservable labor-enhancing variable e_t instead of observable productivity.[13] The sign of $\beta_u/(\alpha - \mu\lambda)$ and thus whether unemployment co-moves with adjusted wages $w_t - e_t$ depends on the substitution elasticity. For example, if labor and capital are gross complements ($\sigma < 1 \Rightarrow \beta_u/(\alpha - \mu\lambda) > 0$), in equilibrium higher unemployment levels would be accompanied by *lower* wages relative to technology and human capital accumulation.

It is also worth acknowledging that this model does not automatically exhibit system stability under exogenous technical progress. The relevant condition on the adjustment coefficient, $-(\alpha - \mu\lambda)/\alpha < 0$, depends on the parameter values, i.e. the system is unstable *iff* $1 < \sigma \leq 1/(1 - \mu\lambda)$.[14] However, endogenizing the labor-enhancing variables e_t –which is perfectly compatible with this model, where e_t is simply left unmodelled– would solve this potential instability (see e.g. Acemoglu, 2003), which in any case appears irrelevant for the United States.

We will assume *balanced growth* regimes in the following sense: the equilibrium growth rates of wages and productivity are assumed to be the same, $\Delta\overline{w} = \Delta\overline{q} = g_{(t)} > 0$, where we have attached a time index in parentheses to reflect the assumption that this steady-state growth rate may change, but only occasionally.[15] This implies that the (log) labor share is not drifting. We saw before that this also implies $\Delta\overline{e} = g_{(t)}$. Unemployment itself will then on average be constant in each regime, therefore $\Delta\overline{u} = 0$. The adjusted wage level $w - e$ is not drifting, but its status is that of an initial condition and we write it accordingly as $(w - e)_0$. Then the balanced growth equilibrium is given by:

$$\overline{u}_{(t)} = \frac{1}{\beta_u}\left(c_{eq}\alpha - \mu\lambda g_{(t)} - (\alpha - \mu\lambda)(w - e)_0\right) \tag{21}$$

[13] In the knife-edge case of Cobb-Douglas production ($\sigma = 1$) this term would reduce to $-(1/\mu\lambda)[\beta_u u_{t-1}]$.

[14] The same coefficient and thus the same condition appears if instead of a wage equation (with Δw_t on the left-hand side) the equation for productivity (with Δq_t on the left-hand side) is used.

[15] In the empirical analysis we identify two such shifts over a span of roughly fifty years.

Thus there are many parameters that determine steady-state unemployment in general, including the intercept of the reservation wage equation (c_r, which is in c_{eq}), product and labor market competitive environments (the respective elasticities are in c_{ld}, which in turn is in c_{eq} as well), and of course the parameters of wage setting and factor substitution.

However, the important additional factor is the balanced growth rate $g_{(t)}$. Except in the extreme case $\mu\lambda = 0$ the equilibrium growth rate of productivity has a negative influence on steady-state unemployment. The cause of this effect is sometimes interpreted as slowly adjusting wage aspirations, because $\lambda \neq 0$ means that wages are partially determined by past wages, instead of being fully determined by contemporaneous productivity.

5 Conclusions

This paper has dealt with the long-run connection between labor productivity growth and unemployment, exploiting the well-known slowdown of productivity growth in the 1970s and the later speed-up in the 1990s, which were treated as exogenous determinants for unemployment developments. Our results indicate that a "co-breaking" framework is a natural and empirically adequate model to capture the long-run link between productivity growth and unemployment in the United States. Such a framework models the non-stationarity in the individual variables through infrequent shifts in their means, i.e. in the so-called deterministic component. These shifts are common to both variables; however, at medium to high frequencies the properties of the variables are quite different, namely high persistence in unemployment and little serial correlation in productivity growth. As a consequence of the *common* shifts there exists a long-run relation as a linear combination of the variables that is free from mean-shifts; our estimates imply a negative long-run connection between productivity growth and unemployment. Unfortunately a necessary condition for a co-breaking analysis as in the present paper is that the number of variables must not exceed the number of breaks ($n \leq m$), which is a severe restriction. But of course it is possible to add stationary variables to a system once co-breaking has been

established.

An additional finding was that the productivity speedup after 1994 restored pre-1970 growth rates (about 3 percent) and therefore also pulled mean unemployment back to its original level (roughly 5 percent). This result suggests that the regime from 1974 to 1994 constituted a historical exception, while the pre-1974 and post-1994 regimes represent the normal workings of the US economy. Greenwood and Yorukoglu (1997) for example have argued that during the slow but steady diffusion of computers and associated business practices through all sectors of an economy the measured productivity growth will be lower as usual until the adoption is complete.[16] Our empirical findings are consistent with this explanation of the productivity slowdown.

Finally it may be worthwhile to point out that our results of significant shifts in these variables also carry implications for other empirical applications. For example, Phillips curves estimates must account for the decline of equilibrium unemployment in the 1990s to avoid mis-specification.

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[16] See, e.g., Jorgenson (2001) and references therein for a detailed account of the underlying evidence.

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